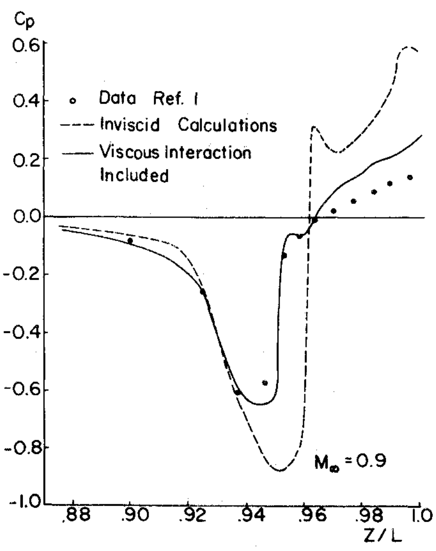

Fig. 3 Results of calculations for $M_\infty = 0.9$.

stretch parameter and r_b is the body profile in the meridian plane. With the proper boundary conditions, the disturbed potential function is calculated through the numerical relaxation scheme by performing finite difference calculations in the $\xi-\zeta$ plane. Different forms of the finite differences representing the partial differentiation in the ξ direction are employed depending upon whether the flow is locally supersonic or subsonic.[2] Final solution is reached when the successive change of the disturbed potential function is within a narrow limit ε (e.g., 3×10^{-6}) throughout the field of calculations. For the viscous flow, it was found that a simple description of the growth of the turbulent boundary- layer in the integral formulation by Sasman and Cresci[8] is adequate, and its effect on the inviscid flow can be accounted for through a displacement thickness correction of the body geometry so that the inviscid calculations are always based on the equivalent inviscid body configurations. Since the inviscid calculations are carried out by repeatedly scanning the field from upstream toward downstream flowfield, it is extremely expedient to perform the viscous calculations at the end of each sweep with the prevailing inviscid flow condition as its guiding freestream. Thus, a new equivalent body shape was established for the next inviscid calculations.

Figure 2 shows the results of such calculations for $M_\infty = 0.8$. The indicated inviscid results were of course obtained by bypassing the viscous flow calculations. It is obvious that the viscous flow effect was playing an equally important role in the solution of the problem and the final results agreed very well with the experimental data. The strong interaction character of the transonic flow past boattailed afterbody is thus fully illustrated.

Similar calculations have been carried out for cases of $M_\infty =$ 0.56 and 0.7 and the final results agreed again very well with the experimental data. For $M_\infty = 0.9$, it was found that "damping" to the change of the equivalent inviscid geometry was necessary before convergent calculations can be achieved. The results for this flow case are shown in Fig. 3. Agreement with the data is reasonably good, although slightly higher pressure level on the last portion of the boattail and a slight overshoot from the shock recompression have been observed.

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Asymptotic Post Buckling Solution of the Ring in an Elastic Foundation

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Introduction

In studying the post buckling of a simply supported strut on a linear Winkler elastic foundation, an interesting and apparently new result was reported in Ref. 1. Depending on certain geometrical parameters, the strut was found to possess either a stable or unstable symmetric point of bifurcation. More recently, it was shown that for the free ends and the semi-infinite simply supported strut only an unstable symmetric bifurcation exists,[2] and consequently imperfection sensitivity can be predicted.[3]

We are, therefore, inclined to ask ourselves whether a similar effect exists for a similar and (moreover) practically important problem like that of an elastically impeded circular ring under external pressure. In the following section we will try to answer this question.

Results of the Analysis

Considering a circular ring of the radius r, the axial stiffness EF, and the bending rigidity EI under a constant directional type of external pressure P, it is easy to show that the potential energy functional is given by

$$V = \int_0^{2\pi} \{\tfrac{1}{2}EI(\dot{\psi}^2/r) + \tfrac{1}{2}EF\varepsilon^2 r + Pr^2[\cos\psi(1+\varepsilon)-1]\}\, d\phi \tag{1}$$

where ε is the axial strain, ψ is the angle of rotation, $(\dot{\ }) = d(\ )/d\phi$ and ϕ is the angular coordinate.

Eliminating the passive function ε, using the equilibrium equation

$$\varepsilon + Pr/EF \cos\psi = 0$$

we obtain

$$V = \int_0^{2\pi} \{\tfrac{1}{2}EI(\dot{\psi}^2/r) + \tfrac{1}{2}(Pr/EF)^2(\cos\psi)^2 r + Pr^2[\cos\psi(1 - Pr/EF\cos\psi)]\}\, d\phi \tag{2}$$

The main difference between Eqs. (1) and (2) is that Eq. (2) is a totally symmetric functional which essentially will facilitate the

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calculation. The following expression, up to third-order terms exact, for the rotation

$$\left(\frac{\psi}{r}\right) = \frac{\dot{w}+v}{r^2(1-Pr/EF)} - \frac{1}{6}\frac{(\dot{w}+v)^3}{r^3(1-Pr/EF)^3} \tag{3}$$

is easily obtained from the differential geometry of the buckled ring and can now be used to write the potential functional Eq. (2) in terms of the radial (w) and the tangential (v) displacement component. Adding to this the energy stored in the assumed Winkler foundation, the total potential functional of the elastically embedded ring is obtained. Expanding the displacement functions as

$$w = \sum_i^{\infty} a_i \cos i\phi \tag{4a}$$

and

$$v = \sum_i^{\infty} b_i \sin i\phi \tag{4b}$$

setting $Pr/EF \cong 0$ and omitting the details of the elementary discrete perturbation procedure,[4] the initial post buckling associated with the smallest critical load (in the case of a relatively large foundation constant c, where large waves number $i \geqq 4$ can occur)

$$P_{\min}{}^c = 2(cEI)^{1/2}/r \tag{5}$$

is given by

$$\frac{P}{P^c} = 1 + \frac{1}{4}\left(\frac{cr^4}{EI}\right)^{1/2} \xi^2 \tag{6}$$

where the associated wave number is $i = r(c/EI)^{1/4}$, $\xi = a_s/r$ is the perturbation parameter and a_s is the maximal buckling amplitude. Unlike the corresponding strut problem, the ring in an elastic foundation possesses a stable symmetric point of bifurcation and is, therefore, imperfection insensitive.

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Two-Dimensional Inverse Conduction Problem—Further Observations

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IN a recent series of papers, Refs. 1 and 2, the author develops an analytical method for temperature extrapolation in heat conduction systems. For the one-dimensional configuration, a solution is generated whereby the differential equation is satisfied in a manner which does not restrict its extrapolation property. From the two prescribed interior conditions

$$T(x_1, t) = T_1(t) \quad \text{and} \quad T(x_2, t) = T_2(t) \tag{1}$$

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the temperature may be predicted outside the spatial range, $x_2 - x_1$. Since it is presumed that Eq. (1) represents thermocouple responses, the actual traces are approximated by two temporal power series of degree, n. Dependent upon the direction of the predication process, one of the interior conditions is reformulated in the Laplace transform plane so that the positive arguments in the solution are suppressed. Therefore, the relationship between the thermocouples is written as

$$\bar{T}_1(s) = \bar{T}_2(s) \sum_{m=1}^{M} A_m \exp\left[-m(x_2 - x_1)(s/\alpha)^{1/2}\right] \tag{2}$$

where s is the complex number and α the thermal diffusivity. The summation in the transformed temperature relationship, Eq. (2), constitutes an enabling function whose coefficients A_m are evaluated from the inverse transform. The explicit expression for the temperature predictor is shown in Ref. 1, and it should be noted that this solution fulfils the requirements that the temperatures must be known throughout a closed interior region. In the one-dimensional case the two planes $x = x_1$ and $x = x_2$ constitute the closed region. Obviously, the solution will return the values for the thermocouple responses dictated by Eq. (1). It is also the solution to the direct conduction problem for a slab of thickness, $x_2 - x_1$, with boundary conditions represented by Eq. (1).

Before proceeding to a discussion of the two-dimensional case, it has been demonstrated that the one-dimensional prediction process is very accurate. Its success may be attributed to the limited number of inter-related approximations that must be made. Experience gained through use of the method indicates that a ninth degree polynomial approximation for Eq. (1) was sufficient. In Eq. (2) any value of M may be selected; however as this value approached the degree of the power series the results were substantially improved. For high accuracy, it is recommended that the value $M = n$ be used. There is little difficulty in the numerical evaluation of the coefficient A_m, since these are obtained from a, $n \times n$, matrix which is insensitive to roundoff errors. As a precaution, sentinel thermocouples may be positioned outside the range, $x_2 - x_1$, and the extrapolation values may be compared with these. The closer the correspondence, the more successful the prediction method.

Considerably more complicated is the extrapolation program for a two-dimensional system. In this instance, the temperature responses are only known at discrete locations around the perimeter of an internally enclosed region. Representing the thermocouple traces as

$$T(x_i, y_j, t) = \sum_{n=1}^{n} b_n{}^{ij} t^n \tag{3}$$

there can be a large number of data points which the analytical solution must agree with. As shown in Ref. 2, a solution is obtained by point-matching. The essence of the method is to satisfy four perimeter positions simultaneously for each increment in time. To illustrate what can occur, suppose the thermocouples are purposefully positioned on an interior rectangle, the thermocouple rectangle. Initially, a solution is obtained with the temperature traces at the corners as the only input data. For this case, the corner solution, the expression not only satisfies the differential equation by construction, but it also returns the values for the temperature traces at the four corners. It does not however follow that the desired solution has been obtained. Additional thermocouple information positioned around the perimeter of the thermocouple rectangle should be compared with the theoretical results. If the resultant deviations are all small, then it can be anticipated that the prediction process will be successful. Positioning of sentinel thermocouples, provides an additional check.

The corner solution involves the least number of approximations, and the related matrix for the determination of the coefficients in the enabling functions is modest in size; consequently the matrix solution is insensitive. It is interesting to note that the corner solution has no parallel as a solution to a direct conduction system. Specifying the boundary conditions